Module 2: Motion In One Dimension

Introduction

As I said in the introduction to Module #1, the science of physics attempts to explain everything that is observed in nature. Now, of course, this is a monumentally impossible task, but physicists nevertheless try to do the best job they possibly can. Over the last 3,000 years, remarkable advances have been made in explaining the nature of the world around us, and in this physics course, we will learn about many of those advances. This module will concentrate on describing *motion*.

If you look around, you will see many things in motion. Trees, plants, and sometimes bits of garbage blow around in the wind. Cars, planes, animals, insects, and people move about from place to place. You should have learned in chemistry that even objects which appear stationary are, in fact, filled with motion because their component molecules or atoms are moving. In short, the world around us is alive with motion.

In fact, St. Thomas Aquinas listed the presence of motion as one of his five arguments for the existence of God. He said that in all of our experience, humans have found that motion cannot occur without a mover. In other words, in order for something to move, there must be something else that moves it. When a rolling ball collides with a toy car, the car will move because the ball gave it motion. But, of course, the ball would not have been rolling to begin with if it had not been pushed or thrown. Thus, Aquinas says that our practical experience says that any observable motion should be traceable back to the original mover. When the universe began, then, something had to be there to start all of the motion that we see today. Aquinas says that God is this "original mover."

While philosophers can mount several objections to St. Thomas Aquinas' argument, it nevertheless shows how important motion is in the universe. Thus, it is important for us to be able to study and understand motion. In this module, we will attempt to understand the most basic type of motion: motion in one dimension. Remember from geometry what "one dimension" means. If an object moves in one dimension, it moves from one point to another in a straight line. In this module, therefore, we will attempt to understand the motion of objects when they are constrained to travel straight from one point to another.

Distance and Displacement

When studying the motion of an object, there are a few very fundamental questions you can ask yourself. You can ask "where is the object," "how fast is it moving," and "how is the object's motion changing?" In physics terms, we say that the answers to these questions are the object's **displacement**, **velocity**, and **acceleration**. We will examine each of these concepts individually, so for right now, let's concentrate on the concept of displacement.

<u>Displacement</u> - The position of an object relative to a fixed point

Before we study this concept in detail, you need to be aware of a very important distinction that physicists make. Notice the definition of displacement. It says that in order to determine displacement, you must determine the position of an object compared to some fixed point in space. Although this might sound pretty basic to you, it is actually one of the fundamental concepts in this module. The best way to show you the importance of the concept is by example.

Suppose you are sitting on the sofa reading a book (maybe even this one), and you suddenly decide that you want to go to the refrigerator for a drink. You get up, and you move to the refrigerator which is 10 meters away from the sofa. You get your drink and then walk 10 meters back to the sofa. How much distance did you travel in your quest for liquid refreshment? Well, you walked 10 meters there and 10 meters back, so you walked a total of 20 meters. After everything was finished, what was your total displacement? *It was zero meters*!! You see, before everything began, you were at the sofa. Since you started there, we can define it as the fixed point. You moved to the refrigerator, at which point you were 10 meters displaced from the sofa. However, when you turned around and came back, you ended up at exactly the same point from which you started. In the end, then, you were 0 meters from the fixed point, thus your displacement was 0.

You see, then, that the concept of displacement carries with it some information about direction, whereas the concept of distance does not. In the situation we just imagined, you walked a *distance* of 20 meters, but your *displacement* was 0 because you walked 10 meters in one direction and then another 10 meters in precisely the opposite direction. Since the displacement in one direction canceled the displacement in the opposite direction, your total displacement was zero. When a physical quantity carries information concerning direction we call it a **vector quantity**. When the physical quantity does not carry information concerning direction, we call it a **scalar quantity**.

Vector Quantity - A physical measurement that contains directional information

Scalar Quantity - A physical measurement that does not contain directional information

Thus, distance is a scalar quantity and displacement is a vector quantity.

When dealing with displacement, we must find some mathematical way to denote the direction that is inherent in the measurement. The way we will do this is to label displacement in one direction positive and displacement in the opposite direction negative. That way, when you add displacements together, motion in one direction will cancel motion in the opposite direction. Thus, we could say that in the situation above, your displacement was +10 meters when you moved from the sofa to the refrigerator and -10 meters when you moved the opposite direction from the refrigerator to the sofa. Your total displacement, then, was +10 meters plus $^{-}$10 meters, which is zero.

What's really nice about this mathematical way of noting direction is that it doesn't really matter which direction you label as positive or which you label as negative. We could just have easily said that your displacement when you arrived at the refrigerator was -10 meters. That would mean that your displacement when you moved from the refrigerator to the couch was +10 meters. The total displacement would still be zero. Thus, it doesn't matter which direction you label as positive, as long as you keep it consistent. Study the following example and solve the "on your own" problem after it to make sure you understand this important principle.

EXAMPLE 2.1

A child is standing 5.0 meters away from a wall and rolls a balls towards it. The ball hits the wall and bounces back, rolling 3.3 meters before coming to a halt. What is the total distance covered by the ball? What is the ball's displacement?

The total distance is easy to calculate. The ball rolled 5.0 meters to reach the wall and 3.3 meters in the other direction after bouncing back. The total distance then, is simply:

$$\text{Total Distance} = 5.0 \text{ meters} + 3.3 \text{ meters} = 8.3 \text{ meters}$$

Calculating the displacement is a bit more difficult, however. To do this, we must first define directions. I will say that motion from the child to the wall represents positive displacement while motion from the wall to the child is negative displacement. Thus, the ball first had a displacement of +5.0 meters and then a displacement of -3.3 meters. The total displacement, then, is:

$$\text{Total Displacement} = 5.0 \text{ meters} + {}^{-}3.3 \text{ meters} = 1.7 \text{ meters}$$

This is a positive displacement, which means that the ball is 1.7 meters away from the child, in the direction of the wall.

Alternatively, I could have said that motion from the child towards the wall represented negative displacement. In that case, the ball would have had a -5.0 meters displacement followed by a +3.3 meters displacement. This would indicate a total displacement of -1.7 meters. You might think that this is a different answer than the one I got previously, because this one is negative. Remember, however, what negative displacement means in this case. It means displacement *from the child towards the wall*. Thus, my answer is still 1.7 meters away from the child, in the direction of the wall. As long as you stay consistent, your answer will be the same regardless of which direction you say is positive and which is negative. The trick is to give your answer in relation to a fixed point, not with just a positive or negative sign.

ON YOUR OWN

2.1 An ant starts at his anthill and walks 15.2 cm to a crust of bread. The ant takes the bread, turns around, and walks back towards his anthill. He stops after he has traveled 3.8 cm and eats part of the crust of bread. What is the total distance he traveled up to that point? What is the total displacement?

Speed and Velocity

Now that you have some idea of what displacement is, you can begin to learn about velocity.

Velocity - The time rate of change of an object's displacement

Now this definition may sound a bit strange, but it is really easy to understand. Velocity simply tells us how quickly an object's displacement is changing. That's what "time rate of change" means. In order to determine this, all you need to do is take the change in displacement and divide it by the time it took to make that change. Mathematically, we could say :

$$v = \frac{\Delta x}{\Delta t} \quad (2.1)$$

where **v** represents the velocity, **x** represents the displacement, and t represents time. The symbol "Δ " represents the Greek letter "delta" and means "change in." Thus, "Δ **x**" means the change in displacement while "Δ t" means the change it time.

There are two very important things you need to learn about Equation (2.1). First, since we calculate velocity by taking displacement (usually measured in meters) and dividing by time (usually measured in seconds), then the standard unit for velocity is meters/second ("meters per second"). Thus, if I travel for

30.0 seconds and my total displacement during that time is 60.0 meters, my velocity is 60.0 meters ÷ 30.0 seconds, or 2.00 meters/second (abbreviated as m/sec). The second thing you need to learn about this equation is that velocity and displacement are both vector quantities. We already learned that about displacement, and since you use displacement to calculate velocity, it only makes sense that velocity is also a vector quantity. This means, then, that whenever you use velocity, you must be sure to keep track of direction. Mathematically, we will do it the same way we did with displacement. Motion in one direction will be noted as positive velocity, while motion in the opposite direction will be written as a negative velocity.

What about time in Equation (2.1)? Is it a vector or a scalar quantity? Well, if you think about it, time only goes one way. As far as we can tell, time cannot go in reverse. Thus, since time does not have a direction attached to it, it is considered a scalar quantity. This is why I have written **v** and **x** in boldfaced type but kept t in normal type. The boldfaced type indicates that **v** and **x** are vector quantities. Since t is not in boldfaced type, you can assume it is not a vector. This kind of notation will exist throughout the rest of the course. When I write a variable in boldfaced type, it will mean that the variable is a vector quantity. If the variable is not in boldfaced type, it will be considered a scalar quantity.

Now it is very important that you do not confuse the concept of velocity with the concept of speed. Just as distance and displacement are different quantities, so are velocity and speed.

Speed - The time rate of change of the total distance traveled by an object

In other words, to determine the speed of an object, you take the total distance traveled and divide by the time it took to travel that distance. Mathematically, we could say:

$$s = \frac{\Delta d}{\Delta t} \quad (2.2)$$

where s represents speed, d represents distance, and t represents time. Notice that none of the variables in this equation are written in boldfaced type. This indicates that there are no vectors in Equation (2.2), and that is the main difference between velocity and speed. While velocity *is* a vector quantity, speed *is not*. Thus, although Equations (2.1) and (2.2) look very similar, speed and velocity are quite different, because one is a vector and one is not. Let's study a couple of examples to make sure we understand these distinctions.

EXAMPLE 2.2

You hop on your bicycle and pedal 151.1 meters to the end of your street in 11.2 seconds. You then turn around and pedal back to where you started. If the return trip takes 15.1 seconds, what was your speed and what was your velocity over the course of the entire bike ride?

We will solve for speed first, because that's a little easier. According to Equation (2.2), we can figure out speed by taking the distance traveled (Δ d) and dividing by the time it took to travel that distance (Δ t). If the street is 151.1 meters long and you traveled to the end and back, you traveled a total distance of 2 x 151.1 m or 302.2 m. The total time it took to travel that distance was 11.2 sec + 15.1 sec or 26.3 sec. thus, according to Equation (2.2):

$$s = \frac{302.2\ \text{m}}{26.3\ \text{sec}} = 11.5\,\frac{\text{m}}{\text{sec}}$$

Calculating velocity, however, is quite another matter. Velocity is determined by taking the total displacement and dividing by the time it took to achieve that displacement. By the time that the bike ride was over, your total displacement was zero, because you ended up back where you started. Thus, Equation (2.1) becomes:

$$v = \frac{0\text{m}}{26.3\text{sec}} = 0\,\frac{\text{m}}{\text{sec}}$$

In the end, then, while your total speed was considerable (11.5 m/sec), your velocity was zero! It might sound strange that you could ride a bike with zero velocity, but once again, remember that velocity is a vector quantity. When your velocity is zero, it means simply that your total displacement was zero. Thus, even though you pedaled a lot, you ended up going nowhere by the end of your ride, so your displacement and velocity were both zero!

A sprinter runs the 200 (2.00×10^2 m) meter dash in 24.00 seconds. He then turns around and walks 15 meters back towards the starting line in order to talk to his coach. Because he is so tired, it takes him 25 seconds to walk that 15 meters. What was the sprinter's velocity during the 200-meter dash? What was his velocity when he walked back to talk to the coach? What was his velocity for the entire trip?

In this case, we are asked only to calculate velocity, so we will only be using Equation (2.1). Once again, we are dealing with vector quantities here, so we must define direction. I will call motion from the starting line to the finish line positive motion. This makes motion from the finish line to the starting line negative motion. The first part of the question asks us to calculate the sprinter's velocity during the 200-meter dash. During that time, the sprinter was moving from the starting line to the finish line. Thus, his displacement was $2.00\text{x}10^2$ meters. It took him 12.00 seconds to make the run, so Equation (2.1) becomes:

$$v = \frac{2.00\times10^2\ \text{m}}{24.00\,\text{sec}} = 8.33\,\frac{\text{m}}{\text{sec}}$$

We could therefore say that his velocity was <u>8.33 m/sec in the direction of the finish line</u>. The second part of the question asks us to calculate his velocity as he is walking back to speak with his coach. During that time, he walked towards the starting line, so his displacement was negative:

$$v = \frac{-15\text{m}}{25\,\text{sec}} = -0.60\,\frac{\text{m}}{\text{sec}}$$

Thus, we could say that his velocity was <u>0.60 m/sec towards the starting line</u>. Finally, the problem asks us to determine his velocity over the entire trip. Well, in order to determine velocity, we must first determine displacement. When the sprinter finished the race, his displacement was 200 meters. However, when he walked back to talk to his coach, his displacement changed by -15 meters. Thus, his total displacement was $2.00\text{x}10^2$ m + -15 m = 185 m. The total time it took to achieve that displacement was 24.00 sec + 25 sec = 49 sec. Equation (2.2), then, becomes:

$$v = \frac{185\text{m}}{49\,\text{sec}} = 3.8\,\frac{\text{m}}{\text{sec}}$$

Since the velocity is positive, we know that even though he walked back a little, his overall velocity was still 3.8 m/sec in the direction of the finish line.

Now you need to answer the following "on your own" problem to make sure you understand these concepts.

ON YOUR OWN

2.2 A mail carrier drives down a street delivering mail. She travels 3.00×10^2 meters down the street in 332 seconds. She then turns around and heads back up the street, but because of the way the mailboxes are placed, she only needs to travel 208 meters in that direction, and that trip takes her only 2.30×10^2 seconds. What was her velocity as she traveled down the street? What was it as she traveled up the street? What was her velocity for the entire trip?

Now, of course, Equation (2.1) has more applications than the ones you have seen so far. Study the next example and solve the "on your own" problem that follows in order to see how other types of problems can be solved using Equation (2.1).

EXAMPLE 2.3

A jogger runs down a long, straight country road at 2.3 m/sec. If she jogs in that direction for 15.3 minutes, how far does she run?

Part of the trick to solving physics problems is learning how to read the question so that you see what you are trying to solve for. In this example, a couple of words should jump out at you. First, you are given a speed, but you are also given direction because the words "straight" and "down" are used. Thus, the 2.3 m/sec is a velocity, because direction is included. The problem also gives you time, but it is not in units that are consistent with the velocity. The velocity is given in m/sec, but the time is given in minutes. To be able to use both of these pieces of information in any solution, the units must be consistent. We therefore must convert one of these quantities into different units. Since m/sec is the standard, I won't convert it. Instead, I will convert 15.3 minutes into seconds:

$$\frac{15.3 \cancel{\text{min}}}{1} \times \frac{60 \text{ sec}}{1 \cancel{\text{min}}} = 918 \text{ sec}$$

Now that we have our units straight, we can continue. The problem wants us to determine how far the jogger will go. Well, "how far" is another way of saying "how much displacement." After all, if she runs, say, 100 meters, her displacement from the place that she started will be 100 meters. Thus, we are given velocity and time and asked to determine displacement. Equation (2.1) relates these three quantities. We will therefore use Equation (2.1), substituting the values that we already know:

$$v = \frac{\Delta x}{\Delta t}$$

$$2.3\ \frac{m}{sec} = \frac{\Delta x}{918\ sec}$$

Now we can use algebra to rearrange this equation and solve for the change in displacement (Δ **x**):

$$2.3\ \frac{m}{\cancel{sec}} \times 918\ \cancel{sec} = \Delta x$$

$$2.1 \times 10^3\ m = \Delta x$$

Since the change in displacement is the same thing as how far the jogger ran, the jogger ran $\underline{2.1 \times 10^3\ m}$.

Do you see how we solved this problem? We read it carefully, and we picked out words that told us what quantities we had and what quantities we needed to determine. We then found an equation that related these quantities and used algebra to solve the equation. This is the way you solve virtually every physics problem known to humankind. Try it yourself on the following "on your own" problem.

ON YOUR OWN

2.3 A boat travels straight down river at a speed of 15 m/sec. If the boat travels a distance of 34.1 km, how long was the boat ride?

Average and Instantaneous Velocity

In "on your own" problem 2.2 and in the example preceding it, we got answers that you might think are a bit strange. In the example, for instance, the sprinter's velocity over the entire trip was 3.8 m/sec in the direction of the finish line. You might find it odd that despite the fact that the sprinter traveled in both directions, his overall velocity was in the direction of the finish line. If you find it strange, don't worry. That's because we haven't discussed the difference between instantaneous and average velocity. We'll do that now.

Instantaneous Velocity - The velocity of an object at one moment in time

Average Velocity - The velocity of an object over an extended period of time

These two concepts of velocity are quite different. To see how different they are, perform the following experiment.

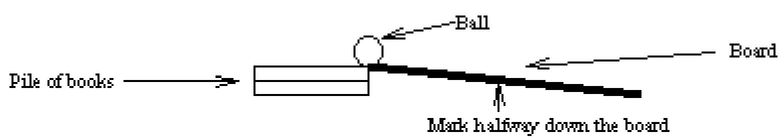
EXPERIMENT 2.1

Measuring Velocity Over Different Time Intervals

Supplies:

- A wooden board, about 1 meter long (Any long, flat surface that you can prop up on one end will do. If you can't find anything, try to find a hill outdoors that is about 1 meter long.)
- A pencil (Anything that you can use to mark the board will do.)
- A stopwatch (A watch with a second hand will do.)
- A pile of books between 6 and 9 centimeters thick
- A ball that will easily roll down the board

Make a mark on the board in the center. Make sure the mark is easy to see. Next, prop the board up on one end with the books, so that the board forms an incline as shown below. In a moment, you will be rolling the ball down the incline. Your experiment should look something like this:



Measure the distance from the top of the board to the mark halfway down the board. You should write your measurement to the nearest 0.01 m. Call this distance "d1." Measure the distance from the mark to the end of the board as well, writing your answer to the nearest 0.01 m. Call it "d2." If you really made the mark in the center of the board, then d1 and d2 should be the same. If not, don't worry about it. They do not have to be equal.

Once you have set your experiment up and made both distance measurements, hold the ball on the very top of the board and be ready to release it. At the exact same moment that you release the ball, start the stopwatch. Stop the watch when the ball hits the mark. Write down the time you measured. Be as precise as possible. If the ball does not want to roll down the board, add another book to the pile to make the incline steeper. Once you get the ball to roll easily, repeat this measurement two more times. After you have three measurements for the time, average them and write down your answer.

As I mentioned in Module #1, you can reduce random errors in experiments by averaging results. That's why we made the same measurement three times and then averaged them. The average of your three measurements is a more accurate determination of the time it took the ball to roll to the first mark. Once you have that average, divide it into the distance from the top of the board to the first mark (d1). Since the ball is always traveling in the same direction, we can say that motion in that direction is positive. That way, the distance we measured will also be the ball's displacement. Thus, the calculation you just made took displacement and divided it by time, which gives you the velocity of the ball as it traveled from the top of the board to the first mark. Call this velocity v1.

Now, hold the ball at the top of the board again, and be ready to release it. This time, however, *do not start the stopwatch until the ball hits the first mark*. Stop the watch when the ball hits the end of the board. Do this measurement three times as well, averaging the results. Then take the average and divide it into d2. This will give you the velocity of the ball as it traveled down the second half of the board. Call it v2. Finally, do the same thing again, this time starting the watch the instant that you release the ball and stopping the watch once the ball hits the end of the board. Average the three results and divide that average into the total length of the board (d1 + d2). This is the velocity of the ball over the entire trip. Call it v3.

Compare your three velocities. If you did the experiment correctly, v1 should be less than v2. V3 should be between v1 and v2. Why? Well, the ball was speeding up the whole time it traveled down the board. Thus, v1 is the lowest because the ball had not sped up all of the way by the time it hit the first mark. V2 was larger than v1 because the ball had more time to speed up traveling down the second half of the board. The total velocity (v3) was the *average* of the two velocities you measured. That's why it falls in between them.

That's the difference between instantaneous and average velocity. You can think of v1 and v2 as *instantaneous* velocities, because they represent the velocity of the ball at different moments throughout its journey. V3, on the other hand, is the *average* velocity, because it represents the velocity of the ball over the entire journey. The average velocity tends to "wash out" differences in the instantaneous velocities. That's why v3 fell in between v1 and v2.

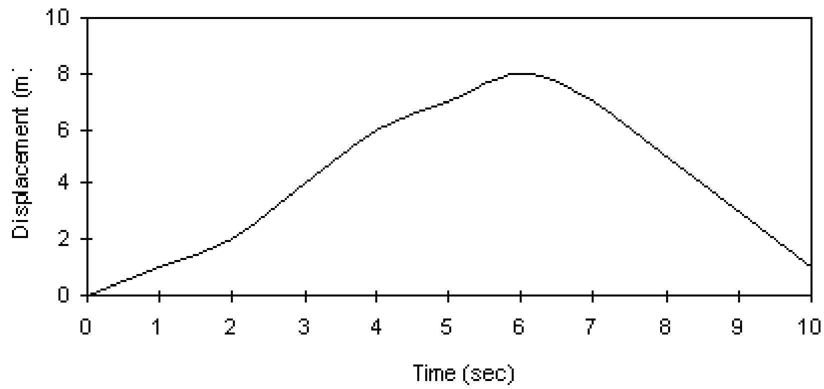
This is why you can get strange answers like the one we got in the last example above. The last velocity that we calculated in the example was the average velocity of the sprinter. This, in effect, averaged the positive and negative velocities that we calculated in the first part of the example. Since the sprinter ran faster and longer in the positive direction, the average velocity turned out to be positive, even though the sprinter traveled in both directions. Thus, average velocity is calculated over a long time span, while instantaneous velocity is calculated over a very short time span.

Now, of course, none of the velocities you calculated in the experiment above were really instantaneous velocities. After all, the instantaneous velocity of an object is the velocity at some exact moment in time. We have no way of measuring that. Instead, the shorter the time interval we use to measure the velocity, the closer we get to the true instantaneous velocity. In other words, if we had put four marks on the board and measured the velocity in four different segments, then each of those velocities would have been closer to real instantaneous velocities than the two that we measured. However, as the time interval gets shorter and shorter, it gets harder and harder to measure velocity. Thus, most instantaneous velocities are, indeed, average velocities, they are just calculated over a short time interval.

Although it is tough to *measure* instantaneous velocities, we can estimate them rather easily by reading a graph. Consider, for example, the graph in Figure 2.1:

FIGURE 2.1

A displacement versus time graph



In this graph, displacement is plotted on the y-axis while time is plotted on the x-axis. Thus, the curve represents an object's displacement at various time intervals. If you look at the graph, you will see that the object starts with zero displacement and then moves in a positive direction to a maximum displacement of about 8 meters. It reaches that maximum displacement in about 6 seconds. At that point, the object's displacement begins to decrease. The only way that can happen is if it begins to move in a negative direction. Thus, after it reached a displacement of 8 meters, the object must have turned around and started moving in the opposite direction.

Now, despite the fact that the velocity is not plotted in this graph, it can be determined. In fact, you can actually get a good feel for the meaning of instantaneous velocity by looking at this graph. I'm getting ahead of myself, however. How can you determine velocity from such a graph? Well, according to Equation (2.2), you can calculate velocity by taking the change in displacement and dividing by the change in time. Well, on this graph, displacement is plotted on the y-axis and time is plotted on the x-axis. Thus, to get velocity, we need to take the change in the y coordinate and divide it by the change in the x coordinate. What's another name for the quantity you get when you take the change in y and divide by the change in x? It's the *slope!* Thus, we come to a very important fact:

The slope of a displacement versus time curve is the velocity.

What does this mean? Well, we can look at the slope of the curve in Figure 2.1 and that represents the velocity of the object. Thus, suppose we looked at the object's displacement at a time of 1.0 seconds. According to the graph, the displacement is about 1.0 m. At 6.0 seconds, however, the displacement is about 8.0 meters. Thus, the velocity during that time interval is:

$$v = \frac{8.0\text{ m} - 1.0\text{ m}}{6.0\text{ sec} - 1.0\text{ sec}} = 1.4\frac{\text{m}}{\text{sec}}$$

On the other hand, suppose we examined the time interval between 0.0 and 1.0 seconds. At zero seconds, the displacement was 0.0, while it was 1.0 m at 1.0 seconds. The velocity over that time frame, then, is:

$$v = \frac{1.0\text{ m} - 0.0\text{ m}}{1.0\text{ sec} - 0.0\text{ sec}} = 1.0\frac{\text{m}}{\text{sec}}$$

Which of those two velocities is closest to an instantaneous velocity? The second one, because it is calculated over a smaller time interval. If we reduced the time interval even more, we would get even closer to a true instantaneous velocity. Taking this reasoning to an extreme, when the time interval is infinitesimally small, the velocity would truly be instantaneous. Thus, if we were to look at a displacement versus time curve at a single point in time, we could estimate the instantaneous velocity by estimating the slope of the curve at that point.

Now if all of this seems a bit confusing, don't worry about it. We'll get lots of practice at examining such graphs, so you'll become a veritable expert at this stuff. Let's look at Figure 2.1 again and look at another time interval. Specifically, let's look at the time interval between 6.0 and 6.2 seconds. What's the velocity during that time interval? Well, according to the graph, the object's displacement seems to stay steady at 8.0 m during that time interval. The velocity, then, is:

$$v = \frac{8.0\,\text{m} - 8.0\,\text{m}}{6.2\,\text{sec} - 6.0\,\text{sec}} = 0.0\,\frac{\text{m}}{\text{sec}}$$

This velocity is close to a truly instantaneous velocity, because the time interval is very short.

In order to make you truly sick of all of this, let's look at one more time interval. What is the average velocity during the time interval of 7.0 seconds to 10.0 seconds? According to the graph, the object's displacement falls from 7.0 m to 1.0 m over that time interval. The velocity, then, is:

$$v = \frac{1.0\,\text{m} - 7.0\,\text{m}}{10.0\,\text{sec} - 7.0\,\text{sec}} = -2.0\,\frac{\text{m}}{\text{sec}}$$

What does the negative sign mean? It means that the object is moving in the opposite direction during this time interval compared to the others we have examined so far. So you see, we can learn a lot about the velocity of an object by looking at a displacement versus time graph.

Now, let's go back to estimating the instantaneous velocity of the object by looking at the graph in Figure 2.1. You should remember from algebra that the steeper a curve rises or falls, the larger its slope is. If the curve rises, its slope is positive, and if the curve falls, its slope is negative. Finally, when the curve is flat, its slope is zero. If we remember these facts, we can answer some pretty fundamental questions about instantaneous velocity when examining a displacement versus time graph.

For example, when does the object reach its maximum velocity? Well, if we look at the figure, the graph seems to be steepest between 3.5 and 4.5 seconds. During that time interval, the velocity is at its maximum value. Now remember, it doesn't matter whether the curve is rising or falling when trying to determine maximum velocity. If the curve happens to be steepest as it is falling, that would be the maximum velocity. Since the negative sign simply tells us direction, we don't consider it when determining where the object is moving the quickest.

Where is the object's velocity at its minimum? Once again, we don't consider negative velocities lower than positive ones, because the negative sign just tells us direction. Thus, the velocity is lowest where the curve is the least steep. That obviously occurs from 6.0 to 6.2 seconds, where the curve is flat.

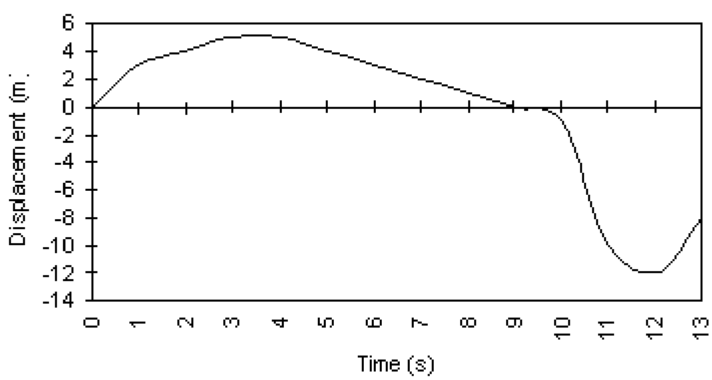
We can also compare instantaneous velocities using the graph in Figure 2.1. For example, which is larger, the instantaneous velocity at 4.5 seconds or the instantaneous velocity at 8.5 seconds? Once again ignoring the positive and negative signs because they simply tell us direction, the curve is obviously steeper at 8.5 seconds than it is at 4.5 seconds. Therefore, the instantaneous velocity of the object is greater at 8.5 seconds than it is at 4.5 seconds.

Sometimes, we can actually give a value for the instantaneous velocity by simply looking at the graph. For example, what is the instantaneous velocity at 6.1 seconds? At that time, the curve is flat. Whenever a curve is flat, its slope is zero. Thus, the instantaneous velocity of the object at 6.1 seconds is zero. Also, consider the time interval between 7.0 seconds and 10.0 seconds. During that time, the curve looks like a straight line. Well, in algebra you should have learned that the slope of a straight line is the same no matter where you are on the line. Since we already calculated that the slope of the curve during this time interval as -1.0 m/sec we can say that the slope of the curve at any point from 7.0 seconds to 10.0 seconds is -1.0 m/sec. Thus, the instantaneous velocity at, say, 8.2 seconds is also -1.0 m/sec.

Since we can learn so much about the velocity of an object from these curves, we need to learn them in detail. Study the next example and then do the "on your own" problem that follows in order to make sure you can interpret graphs like these.

EXAMPLE 2.4

Consider an object in motion whose displacement versus time graph is as follows:



During what time interval is the object moving its fastest?

To answer this question, we simply look for the steepest part of the graph. The curve is clearly steepest from 10 to 11 seconds, so that's the time interval in which the object is moving at its fastest.

How many times does the object change directions?

The slope of the curve starts out positive (because the curve is rising), so the object begins by moving in a positive direction. At around 4 seconds, however, the slope becomes negative (because the curve is falling). This means that the object starts to move in the opposite direction at that time, because the velocity changed direction. That is the first time the object changes direction. At around 12 seconds, the slope changes from negative back to positive, indicating another direction change. Thus, the object changes direction twice.

What is the instantaneous velocity of the object at 12.0 seconds?

At 12.0 seconds, the curve is flat. Thus, the velocity is 0.0 m/sec.

What is the instantaneous velocity of the object at 6.0 seconds?

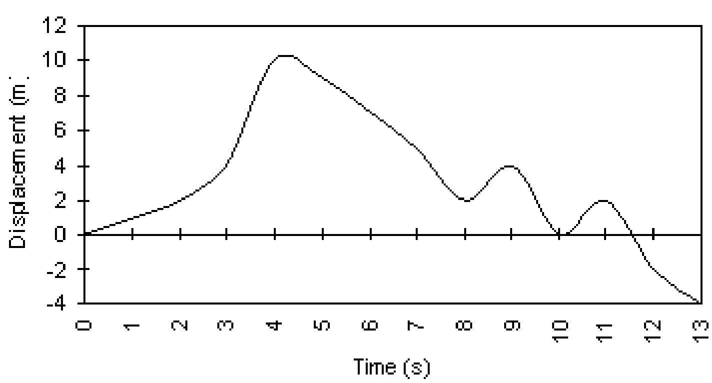
During the interval of 5.0 to 9.0 seconds, the curve looks like a straight line. Thus, the slope at any point along that part of the curve is the same. We can therefore calculate the average velocity from 5.0 to 9.0 seconds and, since the velocity stays the same throughout that time interval, that will also be the instantaneous velocity at any time during that interval, including 6.0 seconds. The displacement at 5.0 seconds looks to be 4.0 meters according to the graph. At 9.0 seconds, the displacement is 0.0 meters. The average velocity, then, is:

$$v = \frac{0.0\,\text{m} - 4.0\,\text{m}}{9.0\,\text{sec} - 5.0\,\text{sec}} = -1.0\,\frac{\text{m}}{\text{sec}}$$

This means that the instantaneous velocity at 6.0 seconds is also -1.0 m/sec.

ON YOUR OWN

Consider the following displacement versus time curve:



2.4 Is the object moving faster at 3.5 seconds or at 8.5 seconds?

2.5 How many times does the object change directions?

2.6 What is the instantaneous velocity at 1.0 seconds?

Before we leave this section, I must point something out. Oftentimes when students are studying introductory physics, they think that the problems they work out are useless exercises. Nothing could be further from the truth! Nearly every aspect of physics has practical applications. Students, however, are not knowledgeable enough realize what they are. For example, students often complain that the displacement versus time graphs we just learned are a waste of time because there are no practical uses for them. How wrong these students are!

Race car drivers spend hours of time on the track, trying to determine the best way to negotiate the curves and straight-aways to get the best time possible. It turns out that while they are on the track, computers keep measuring the car’s displacement and time. At the end of the run, the driver and his team study the displacement versus time graph very carefully. You see, by looking at the slope of the curve, the driver can easily see where the car slowed and where it sped up. If the car was slowing down in the wrong place, studying the displacement versus time curve will show that, and the driver can adjust his or her strategy accordingly. So, now that you understand these displacement versus time curves, you could help a race car driver develop strategies for his or her next race!

Velocity is Relative

Now if all of this velocity talk hasn’t been confusing enough, there is one more concept that we must cover. One of the most important things to realize about velocity is that it is *relative*. What does that mean? The best way to illustrate how velocity is relative is by considering an example. Let’s suppose you’ve just finished visiting your grandmother’s house, and you get in the family car to drive away. You are lucky enough to be riding in the passenger’s seat next to your father, who is driving. Your grandmother, sorry to see you go, has come out of the house and is standing in front of the car waving good-bye. As the car backs out of the driveway, you are looking at your grandmother, waving good-bye as well. Now, answer this one simple question: Are you moving?

Your first instinct is probably to say, "Well yes, of course I’m moving, because the car is backing out of the driveway!" Wait a minute, though. Aren’t you actually sitting still? If your father looks at you, does he think that you’re moving? Probably not. After all, as far as he can see, you are sitting still right next to him. You don’t seem to move at all. From your grandmother’s point of view, however, you are moving. You are moving away from her. That’s the point. As far as your father is concerned, you don’t seem to be moving at all. From your grandmother’s point of view, however, you are, indeed moving. Thus, your father seems to think that your velocity is zero, while your grandmother sees that you have a velocity that is greater than zero and moving away from her.

This is what we mean when we say that velocity is relative. It depends on who is observing that velocity. Since your father is in the same car as you, you are both moving with the exact same velocity. As a result, your displacement relative to him never changes. When displacement doesn’t change, velocity is zero. Thus, your father thinks that your velocity is zero. On the other hand, your displacement relative to your grandmother is increasing. As a result, she sees a velocity greater than zero, moving away from her. Thus, velocity can only be determined relative to an observer.

What an observer actually sees is the difference between his velocity and the velocity of what he is observing. Let’s go back to the situation we were just dealing with. Your grandmother was standing still. Her velocity, then, was zero. You, on the other hand, were moving away from her in the car. The velocity she saw was the difference between her velocity (0) and your velocity. Thus, she observed you moving. Your father, however, was moving with the car and had exactly the same velocity that you did. The difference between his velocity and your velocity, then, was zero, and that’s why from your father’s viewpoint, you were not moving. See if you understand this concept by studying the following example and performing the "on your own" problem afterwards.

EXAMPLE 2.5

A car and a truck are approaching each other on a 2-lane road (see diagram below). A hitchhiker who is standing still on the side of the road is watching them. The speedometer in the car reads 55 mph and the speedometer in the truck reads 45 mph. What velocity does the hitchhiker observe for the car? What velocity does he observe for the truck? What velocity does the driver of the car observe for the truck? What velocity does the driver in the truck observe for the car?

The hitchhiker is standing still, so his velocity is zero. If we define motion to our right as positive, he sees the car moving at 55 mph - 0 mph = 55 mph. Thus, the car is moving to our right at 55 mph according to the hitchhiker. Since the truck is moving to our left, the hitchhiker sees its velocity as -45 mph - 0 mph = -45 mph. As a result, he sees the truck traveling at 45 mph to our left. The driver in the car, however, is already moving. As he looks at the truck, he has no idea what its speedometer reads. What he does see, however, is that the truck is approaching very quickly. As all observers do, he sees the difference between his velocity and the truck's velocity. Thus, the velocity he observes is -45 mph - 55 mph = -100 mph. According to our definition of positive and negative, this means that the car observes the truck moving to our left at 100 mph. Finally, the truck also observes the difference between the car's velocity and his velocity. Thus, the truck observes a velocity of 55 mph - (-45 mph) = 100 mph. The positive sign means that the motion is to our right. Thus, the truck sees the car moving to our right at 100 mph.

In the end, then, the velocity of the car and truck depend on the observer. The hitchhiker observed one set of velocities, while the drivers observed another. As a point of mathematical clarification, when you are calculating the difference in velocities, always take the velocity of what you are observing minus the velocity of the observer. That way, your signs will always work out to the proper directions.

ON YOUR OWN

2.7 A boat is traveling up a river against the current. A boy on a raft is floating down the river with the current. They are both being observed by a fisherman sitting on the shore. The fisherman observes the boat traveling 15 m/sec up river. He also notices that the boy and his raft have a velocity of 3 m/sec down river. What is the velocity of the raft as observed by someone on the boat? What is the velocity of the boat as observed by the boy on the raft?

Acceleration

We now come to the last concept we need to cover in this module: **acceleration**.

Acceleration - The time rate of change of an object's velocity

Does this definition sound similar to the one for velocity? It should. Just as velocity measures how an object's displacement varies with time, acceleration measure how an object's velocity changes with time. The mathematical definition of acceleration is as follows:

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} \quad (2.3)$$

where **a** is the acceleration, **v** is the velocity, and t is time. Once again, since **a** and **v** are in boldfaced type, they are vector quantities.

What units are attached to acceleration? Well, we already know that velocity has the standard unit of m/sec. In order to get acceleration, you take the change in velocity (which still has units of m/sec) and divide by time (which has the standard unit of seconds). What happens when you take m/sec and divide by sec? You get m/sec^2 (meters per second squared). This is the standard unit for acceleration.

Since Equation (2.3) tells us that acceleration is a vector, we need to be sure we understand all of the implications of this fact. When you hear the term "acceleration" in everyday language, it means "speed up." For example, when a driver increases the velocity of a car, we say that the car accelerated. In physics, though, acceleration does not have to mean "speed up." It can also mean "slow down." After all, acceleration just tells us how the velocity of an object is changing. If the velocity is decreasing, then it is changing, and thus there is acceleration.

When we see acceleration, then, how will we know whether it is causing an increase in velocity (speeding the object up) or a decrease in velocity (slowing the object down)? Actually, it is quite simple. If the acceleration and velocity have opposite signs, the object is slowing down. If they have identical signs, the object is speeding up. Thus, if an object has a velocity of - 3.2 m/sec and an acceleration of 0.1 m/sec^2, then the object is slowing down. Alternatively, a velocity of 13.2 m/sec and an acceleration of 2.2 m/sec^2 mean that the object is speeding up. That's the vector nature of acceleration. If acceleration and velocity have the same direction, the acceleration is increasing the velocity. Alternatively, if the acceleration and velocity are pointed in opposite directions, the acceleration is decreasing the velocity. Perform the following experiment to help you understand what acceleration is all about.

EXPERIMENT 2.2

Measuring an Object's Acceleration

Supplies:

- A wooden board, about 1 meter long (Any long, flat surface that you can prop up on one end will do. If you can't find anything, try to find a hill outdoors that is about 1 meter long.)
- A stopwatch (A watch with a second hand will do.)
- A pile of books between 6 and 9 centimeters thick
- A few extra books
- A piece of masking tape or electrical tape
- A ball that will easy roll on the board
- An uncarpeted floor to set the experiment on

Construct the same experimental setup that you had for Experiment 2.1. This time, however, use the tape to make a mark on the floor exactly 1.00 meter from the end of the board. When you have the experiment set up, hold the ball at the top of the board again and release it. Do not start the stopwatch until the instant that the ball rolls off of the board and onto the floor. Stop the watch when the ball reaches the tape. In this way, you have measured the time it takes for the ball to roll one meter once it has left the end of the board. Just as you did in Experiment 2.1, make this measurement three times and average the result. Take that average and divide it into 1.00 m. This, then, measures the velocity of the ball once it rolls off of the board.

If you think about it, the ball rolls down the board because of gravity. We'll discuss that subject several times throughout the course of these modules, so I don't want to talk about gravity itself in depth at this time. Nevertheless, we should all be aware that the reason the ball rolls down the board is that gravity is pulling it down. Since gravity is pulling down on the ball, the ball accelerates. It starts with a velocity of zero (because you held it still to being with), and it rolls off of the board with a large velocity. Since velocity changed, by definition, there must have been acceleration. Gravity supplies that acceleration.

Once the ball leaves the board, however, gravity can no longer accelerate it. Therefore, the ball rolls across the floor with a relatively constant velocity. Now, of course, the ball eventually slows down and stops because it either runs into something or because of *friction*, which we will explore in a later module. For the first meter after it rolls off the board, however, it is a reasonably good assumption that the ball rolls with a constant velocity, as long as the floor that you set the experiment on is not carpeted. Thus, the velocity that you measured is approximately the same as the velocity the ball had when it rolled off the end of the board.

Now, hold the ball at the top of the board again and release it. This time, start the watch as soon as you release the ball and stop it when the ball reaches the end of the board. Once again, make this measurement three times and average the result. Do not calculate any velocities. You are only measuring time in this portion of the experiment. What does this measurement represent? Well, it represents the time it takes for the ball to roll down the board. What's so important about that? Think about it. The ball started (at the top of the board) with a velocity of zero and ended (at the bottom of the board) with the velocity that you measured in the first part of this experiment. Thus, it must have accelerated. When did that acceleration take place? When the object was on the board. Remember, the velocity of the ball stayed constant once it rolled off of the board. This means that all of its acceleration took place while it was on the board. Therefore, we know the beginning velocity (0), and the ending velocity (the velocity that you measured in the first part of this experiment). If we subtract the former from the latter, we will get $\Delta \mathbf{v}$, the change in velocity while the ball was on the board. The time that you just measured is the time interval over which the ball stayed on the board, or Δt. Take your value for $\Delta \mathbf{v}$ and divide it by Δt, and you get the acceleration that the ball experienced!

Now, add 6 - 9 more centimeters of books to the book pile so that the board tilts more steeply. Repeat the entire experiment, so that you get a new value for acceleration. Compare this value with the old one. What happened? When the board was tilted more, the acceleration increased. This should make sense to you. After all, the steeper the hill, the faster the ball should roll. Confirm this concept with one last trial. Add another 6-9 cm worth of books to the pile and do the experiment again. The acceleration that you get should be the largest of the three.

So we see that acceleration is the agent through which velocity change occurs. Study the following examples and solve the "on your own" problems that appear afterward so that you are sure to have a firm grasp of the concept of acceleration.

EXAMPLE 2.6

A car is moving with a velocity of 25 m/sec. The driver suddenly sees a deer in the middle of the road and slams on the brakes. The car comes to a halt in 2.1 seconds. What was the car's acceleration?

This problem is a straightforward application of Equation (2.3). The problem says that the car starts with a velocity of 25 m/sec and ends up stopping (v = 0). Thus, we can subtract the initial velocity from the final velocity to get Δ **v**:

$$\Delta \mathbf{v} = \mathbf{v}_{final} - \mathbf{v}_{initial} = 0 \text{ m/sec} - 25 \text{ m/sec} = -25 \text{ m/sec}$$

The problem also gives us time, so to calculate the acceleration, all we have to do in plug these numbers into Equation (2.3):

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{-25\ \frac{m}{sec}}{2.1\ sec} = -12\ \frac{m}{sec^2}$$

Thus, the car's acceleration was -12 m/sec^2. What does the negative mean? Well, the initial velocity was positive. This means that acceleration and velocity had opposite signs, which means the car was slowing down.

In the next module, we will learn that when objects are dropped, they fall straight down with an acceleration of 9.8 m/sec^2. If a ball is dropped with no initial velocity, how long would it take to accelerate to a downward velocity of 10.0 m/sec?

This problem tells us acceleration and the change in velocity. The velocity starts at 0 and ends at 10.0 m/sec. If we subtract initial velocity (0) from final velocity (10.0 m/sec), we get a Δ **v** of 10.0 m/sec downwards. Since we have acceleration and Δ **v**, then we can use Equation (2.3) to solve for time:

$$a = \frac{\Delta v}{\Delta t}$$

$$9.8\ \frac{m}{sec^2} = \frac{10\ \frac{m}{sec}}{\Delta t}$$

$$\Delta t = \frac{10\ \frac{\cancel{m}}{\cancel{sec}}}{9.8\ \frac{\cancel{m}}{sec^{\cancel{2}}}} = 1.0\ sec$$

Thus, it takes the ball 1.0 second to accelerate to a velocity of 10 m/sec downwards. Now 10 m/sec is about the same as 25 mph, so things that fall speed up pretty quickly!

ON YOUR OWN

2.8 A sprinter starts from rest and, in 3.4 seconds, is traveling with a velocity of 16 m/sec. What is the sprinter’s acceleration?

2.9 A race car accelerates at -7.2 m/sec^2 when the brakes are applied. If it takes 3.1 seconds to stop the car when the brakes are applied, how fast was the car originally going?

Average and Instantaneous Acceleration

Since the equations for velocity and acceleration are similar, you might expect that acceleration, like velocity, can be defined as an average or as instantaneous. Just like velocity, when the time interval is large, the acceleration is an average. When the time interval is very short, however, you can assume that the acceleration is instantaneous. Just like velocity, the only real way to determine instantaneous acceleration is by studying graphs.

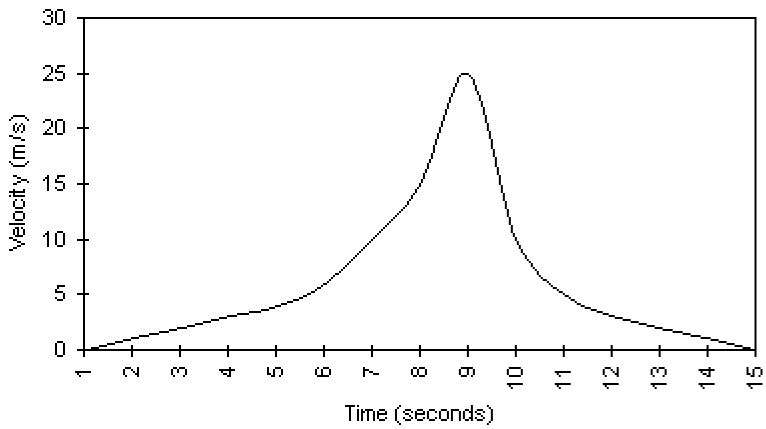
What kinds of graphs will we study in this case, however? Well, since acceleration tells us how velocity changes with time, we should examine velocity versus time graphs. If we plot velocity on the y-axis and time on the x-axis, then the acceleration will end up being the slope of the curve.

Acceleration is the slope of a velocity versus time curve

Since the methods for studying velocity versus time curves are identical to the ones we used to analyze displacement versus time curves, I will not explain them all over again. Instead, study the next example and solve the "on your own" problems that follow to make sure you can analyze these graphs as well.

EXAMPLE 2.7

A race car’s motion is given by the following graph:



Over what time interval is the car speeding up?

The car speeds up when acceleration and velocity have the same sign. According to the graph, velocity is always positive. This means that in order to be speeding up, the acceleration must also be positive. Thus, the curve must always be rising. This occurs during the time interval of 1 to 9 seconds. The car is slowing down from 9 to 15 seconds.

When is the car's acceleration zero?

The slope of a curve is zero when the curve is flat. This happens briefly at 9 seconds.

What is the instantaneous acceleration of the car at 3 seconds?

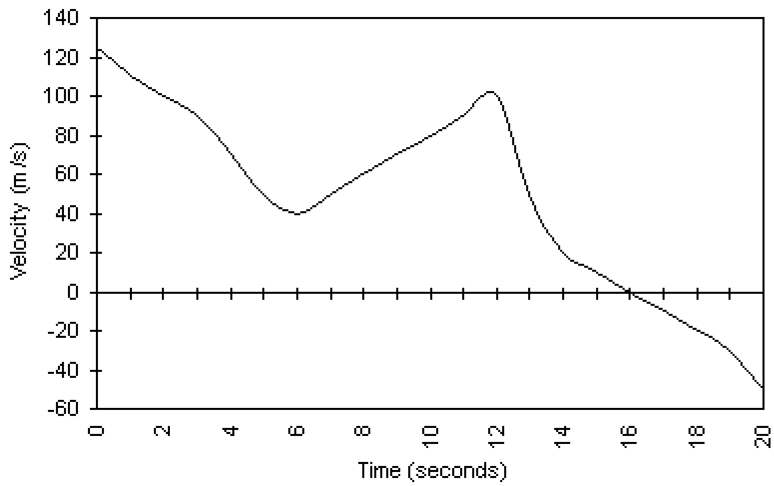
The curve looks like a straight line from 1 to 4 seconds. Thus, the slope of the curve at any point during that time interval is the same as the average slope. At 1.0 second, the velocity is 0.0 m/sec. At 4.0 seconds, the velocity is 4.0. The average slope, then, is:

$$\text{slope} = \frac{4.0\ \text{m/sec} - 0.0\ \text{m/sec}}{4.0\ \text{sec} - 1.0\ \text{sec}} = 1.3\ \text{m/sec}^2$$

This slope is the same throughout that entire time interval, so at 3 seconds, the acceleration is 1.3 m/sec^2.

ON YOUR OWN

Consider an object whose motion is described by the following graph:



2.10 During what time intervals is the object speeding up?

2.11 When is the object's acceleration zero?

Before we finish this module, I need to make two points. First, there is one special property of a velocity versus time curve. The area under such a curve represents the object's displacement. Thus, if I could take the velocity versus time curve above and somehow calculate how much area exists under the line, I would be able to determine the final displacement of the object. Now, of course, you have no way of doing this, so you don't have to worry. I won't ask you any questions about this. It turns out, however, that the mathematical field of calculus is devoted to two things: calculating the slope of curves and *the area under curves*. Thus, when you learn calculus, you will learn another way to analyze these graphs.

The last point I need to make is rather important. If you solved "on your own" problem 2.11 correctly, you found that there were two times that the object had zero acceleration: approximately 6 seconds and 12 seconds. What was the object's velocities at those two times? They were 40 m/sec and 100 m/sec, respectively. Notice that *even though the acceleration was zero at these two times, the velocity was not*. This is an important point and cannot be over-emphasized. It is very tempting to say that velocity is zero when acceleration is zero. Although that is, indeed, possible, it is *not necessary*.

The converse to this statement is just as true and just as important. In the "on your own" section above, what was the velocity of the object at 16 seconds? It was zero. Was the acceleration zero? No, it was negative. We see, then, that acceleration does not have to be zero when the velocity is zero. Acceleration is the *change in* velocity. Thus, it is very possible for one to be zero and the other to be non-zero.

If velocity is zero, acceleration does not have to be zero. If acceleration is zero, velocity does not have to be zero.

Plant this fact in your head, or you will be really lost in the next module!

ANSWERS TO THE ON YOUR OWN PROBLEMS

2.1 The total distance is easy to calculate. The ant crawled 15.2 centimeters in one direction and 3.8 centimeters in the other. The total distance then, is simply:

Total Distance = 15.2 cm + 3.8 cm = 19.0 cm

Calculating the displacement is a bit more difficult, however. To do this, we must first define directions. I will say that motion from the anthill to the bread represents positive displacement while motion from the bread to the anthill is negative displacement. Thus, the ant first had a displacement of +15.2 cm and then a displacement of -3.8 cm. The total displacement, then, is:

Total Displacement = 15.2 cm + -3.8 cm = 11.4 cm

This is a positive displacement, which means that the ant is 11.4 cm away from the anthill. in the direction of the bread.

Note that saying 11.4 cm isn't good enough. With the opposite definition of positive and negative displacement, another person would have gotten -11.4 cm. Both answers would be correct, depending on your initial definition. Thus, we must give the answer in relation to the points in the problem, so that the answer is independent of our definition of positive and negative direction.

2.2 In this problem, we are asked to calculate velocity, so we will be using Equation (2.1). Once again, we are dealing with vector quantities here, so we must define direction. I will call motion down the street positive motion and motion up the street negative. The first part of the question asks us to calculate the mail carrier's velocity while she travels down the street. Well, during that time, her displacement was 3.00×10^2 meters. It took her 332 seconds to travel down the street, so Equation (2.1) becomes

$$v = \frac{3.00 \times 10^2 \text{ m}}{332 \text{ sec}} = 0.904 \frac{\text{m}}{\text{sec}}$$

We could therefore say that her velocity was 0.904 m/sec down the street. The second part of the question asks us to calculate her velocity as she is traveling up the street. During that time, her displacement was negative:

$$v = \frac{-208 \text{ m}}{2.30 \times 10^2 \text{ sec}} = -0.904 \frac{\text{m}}{\text{sec}}$$

Thus, we could say that her velocity was 0.904 m/sec up the street. Finally, the problem asks us to determine her velocity over the entire trip. Well, in order to determine velocity, we must first determine displacement. The mail carrier's total displacement was 3.00×10^2 m + -208 m = 92 m. The total time it took to achieve that displacement was 332 sec + 2.30×10^2 sec = 562 sec. Equation (2.2), then, becomes:

$$v = \frac{92 \text{ m}}{562 \text{ sec}} = 0.16 \frac{\text{m}}{\text{sec}}$$

Since the velocity is positive, we know that even though the mail carrier traveled in both directions, her overall velocity was 0.16 m/sec down the street.

2.3 The problem gives us a speed and a direction. This means that the 15 m/sec is actually a velocity. In addition, we are told how far the boat travels (34.1 km). If we consider the place the boat started as our point of reference, then this distance is actually the change in the boat's displacement during the boat ride (Δ **x**). The problem, however, is that the units do not match. Velocity is in m/sec while displacement is in km. We need to gets these units into agreement, so I will convert km into m:

$$\frac{34.1\ \cancel{km}}{1} \times \frac{1{,}000\ m}{1\ \cancel{km}} = 3.41 \times 10^4\ m$$

Now we can substitute into Equation (2.1), use algebra to rearrange the equation, and solve for time:

$$v = \frac{\Delta x}{\Delta t}$$

$$15\ \frac{m}{sec} = \frac{3.41 \times 10^4\ m}{\Delta t}$$

$$\Delta t = \frac{3.41 \times 10^4\ \cancel{m}}{15\ \frac{\cancel{m}}{sec}} = 2.3 \times 10^3\ sec$$

Thus, the boat ride took 2.3×10^3 seconds, or 38 minutes.

2.4 The slope of the curve is steeper at 3.5 seconds than at 8.5 seconds, so the object is moving faster at 3.5 seconds.

2.5 The slope changes from positive to negative at about 4.0 seconds. This represents one direction change. The slope changes from negative to positive at around 8 seconds, representing the second direction change. It changes from positive to negative at about 9.0 seconds and then again from negative back to positive at about 10.0 second. These represent the third and fourth direction changes. Finally, at 11.0 seconds, the slope changes from positive to negative. This is the fifth (and last) direction change. Thus, the object changed directions 5 times.

2.6 During the interval of 0.0 to 2.0 seconds, the curve looks like a straight line. Thus, the slope at any point along that part of the curve is the same. We can therefore calculate the average velocity from 0.0 to 2.0 seconds and, since the velocity stays the same throughout that entire time interval, it will also be the instantaneous velocity at any time during that interval, including 1.0 seconds. The displacement at 0.0 seconds is 0.0 meters according to the graph. At 2.0 seconds, the displacement is about 2.0 meters. The average velocity, then, is:

$$v = \frac{2.0\,m - 0.0\,m}{2.0\,sec - 0.0\,sec} = 1.0\,\frac{m}{sec}$$

This means that the instantaneous velocity at 1.0 second is also 1.0 m/sec.

2.7 Since the fisherman's velocity is zero, we can use the velocities that he observes to determine the velocities of the raft and the boat relative to each other. We will say that upstream motion is positive and downstream motion is negative. Thus, the boat is traveling at 15 m/sec and the raft is traveling at -3 m/sec. To determine the velocity of an object relative to another, we take the velocity of the thing being observed and subtract from it the velocity of the observer. Therefore, a person on the boat observes the raft moving at -3 m/sec - 15 m/sec = -18 m/sec. Since negative means downstream motion, the people on the boat observe the raft moving 18 m/sec downstream. The boy on the raft, however, observes the boat moving at a velocity of 15 m/sec - (-3 m/sec) = 18 m/sec. Since positive motion is upstream, the boy observes the boat moving 18 m/sec upstream.

2.8 This problem is a straightforward application of Equation (2.3). The problem says that the sprinter starts from rest (v = 0) and sprints to a velocity of 16 m/sec. Thus, we can subtract the initial velocity from the final velocity to get $\Delta \mathbf{v}$:

$$\Delta \mathbf{v} = \mathbf{v}_{\text{final}} - \mathbf{v}_{\text{initial}} = 16 \text{ m/sec} - 0 \text{ m/sec} = 16 \text{ m/sec}$$

The problem also gives us time, so to calculate the acceleration, all we have to do is plug these numbers into Equation (2.3):

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{16\ \frac{\text{m}}{\text{sec}}}{3.4\ \text{sec}} = 4.7\ \frac{\text{m}}{\text{sec}^2}$$

Thus, the sprinter's acceleration was 4.7 m/sec^2. Since acceleration and velocity have the same signs, we know that the sprinter was speeding up.

2.9 This problem tells us acceleration, time, and final velocity and asks us to calculate the initial velocity. We can do this by calculating $\Delta \mathbf{v}$. We can use Equation (2.3) to do this:

$$a = \frac{\Delta v}{\Delta t}$$

$$-7.2\ \frac{\text{m}}{\text{sec}^2} = \frac{\Delta v}{3.1\,\text{sec}}$$

$$\Delta v = -7.2\ \frac{\text{m}}{\text{sec}^{2}} \times 3.1\,\text{sec} = -22\ \frac{\text{m}}{\text{sec}}$$

Now that we have Δ **v**, we can use the definition of Δ **v** to solve for the initial velocity:

$$\Delta \mathbf{v} = \mathbf{v}_{final} - \mathbf{v}_{initial}$$

$$-22 \text{ m/sec} = 0 \text{ m/sec} - \mathbf{v}_{initial}$$

$$\mathbf{v}_{initial} = 22 \text{ m/sec}$$

Thus, the car was originally traveling at 22 m/sec. Notice that the velocity and acceleration have different signs. They should, since the car slowed down!

2.10 Be very careful solving this one. Remember, the object will speed up whenever velocity and acceleration have the same signs. From the time interval of 6.0 seconds to 12.0 seconds, the velocity is positive and the acceleration (slope) is positive. Thus, the object speeds up in that interval. You might be tempted to say that this is the only interval in which the car speeds up, but you would be wrong. From 16.0 seconds to 20.0 seconds, the velocity and acceleration are both negative. Thus, the object is speeding up then as well. Therefore, there are two time intervals during which the object speeds up, 6.0 - 12.0 seconds and 16.0 - 20.0 seconds.

2.11 The acceleration is zero wherever the curve is flat. That happens at about 6.0 seconds and 12.0 seconds.

REVIEW QUESTIONS

1. What is the main difference between a scalar quantity and a vector quantity?

2. On a physics test, the first question asks the students to calculate the acceleration of an object under certain conditions. Two students answer this question with the same number, but the first student's answer is positive while the second student's answer is negative. The teacher says that they both got the problem 100% correct. How is this possible?

3. Which is a vector quantity: speed or velocity?

4. What is the main difference between instantaneous and average velocity?

5. What physical quantity is represented by the slope of a displacement versus time graph?

6. What do physicists mean when they say that velocity is "relative?"

7. You are reading through someone else's laboratory notebook, and you notice a number written down: 12.3 m/sec^2. Even though it is not labeled, you should immediately be able to tell what physical quantity the experimenter measured. What is it?

8. Another experiment in the same laboratory notebook says that an object has a 1.4 m/sec^2 acceleration

when it has a -12.6 m/sec velocity. At that instant in time, is the object speeding up or slowing down?

9. What kinds of graphs do you study if you are interested in learning about acceleration?

10. An object's velocity is zero. Does this mean its acceleration is zero? Why or why not?

PRACTICE PROBLEMS

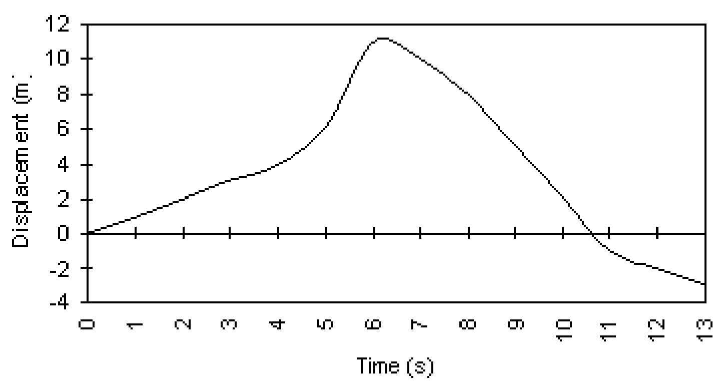
1. A delivery truck travels down a straight highway for 35.4 km to make a delivery. On the way back, the truck has engine trouble and the driver is forced to stop and pull off the road after traveling only 13.2 km back towards its place of business. How much distance did the driver cover? What is his final displacement?

2. If the driver in the above problem took 21.1 minutes to reach the delivery point and broke down 7.5 minutes into the return trip, what was the average speed? What was the driver's average velocity?

3. A plane flies straight for 672.1 km and then turns around and heads back. The plane then lands at an airport that is only 321.9 km away from where the pilot turned around. If the plane's average velocity over the entire trip was 42 m/sec, how much time did the entire trip take?

Questions 4 - 6 refer to the figure below

A car's motion is described by the following displacement versus time curve:



4. At approximately what time does the car change its direction?

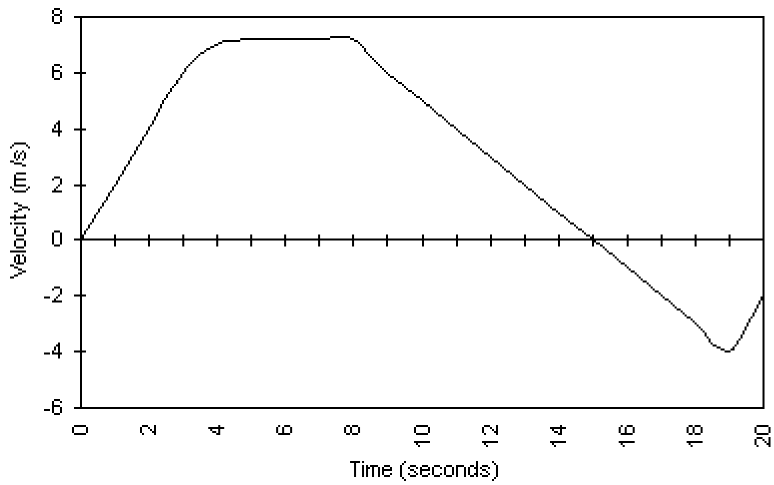
5. Over what time interval is the car moving the fastest?

6. Is the car moving faster at 4.0 seconds or at 10.5 seconds?

7. A train is traveling with an initial velocity of 20.1 m/sec. If the brakes can apply a maximum acceleration of -0.0500 m/sec^2, how long will it take the train to stop?

Questions 8 - 10 refer to the figure below

A runner's motion is described by the following velocity versus time graph:



8. Over what time intervals is the runner slowing down?

9. What is the runner's acceleration at 6.0 seconds?

10. What is the runner's acceleration at 2.2 seconds?